

Electric Fields in a Tokamak

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Abstract

With the establishment of vanishing net electrostatic fields in a toroidally symmetric tokamak at equilibrium [R. W. Johnson, *to appear in Phys. Rev. D*], one is left needing an explanation for the measurement of an apparent radial electric field in experiments. Two scenarios are proposed, depending on the type of measurement being considered. Indirect measurement via the radial equation of motion for an impurity species possibly measures that species' net radial viscous force, and direct measurement via the motional Stark effect might reveal electric fields generated by the shifting of the toroidal magnetic flux density.

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I. INTRODUCTION

With the establishment of vanishing net electrostatic fields in a toroidally symmetric tokamak at equilibrium [1], one is left needing an explanation for the measurement of an apparent radial electric field in experiments. Two scenarios are proposed, depending on the type of measurement being considered. Indirect measurement via the radial equation of motion for an impurity species possibly measures that species' net radial viscous force, and direct measurement via the motional Stark effect might reveal electric fields generated by the shifting of the toroidal magnetic flux density.

II. INDIRECT MEASUREMENT

The indirect measurement of the radial electric field [2] is accomplished by evaluating the RHS of the charge density normalized radial equation of motion for an impurity species s , usually carbon,

$$\tilde{E}_r = B_\theta V_{\phi s} - B_\phi V_{\theta s} + p'_s/n_s e_s, \quad (1)$$

for $p'_s \equiv \partial p_s / \partial r$. We consider a concentric circular flux-surface geometry (r, θ, ϕ) , Figure 1 in Reference [3], related to cylindrical coordinates (Z, R, ϕ) by $R = R^0 + r \cos \theta \equiv R^0(1 + \epsilon \cos \theta)$, $Z = -r \sin \theta$, $\phi = \phi$, where here $\hat{\phi}$ is taken along the direction of the toroidal magnetic field, and assume toroidal symmetry, $\partial/\partial\phi \equiv 0$. The evaluation is performed using flux-surface averaged values, resulting in a nonzero value for the net radial electric field. Returning to the radial equation of motion, however, we find that

$$n_s m_s [(\mathbf{V}_s \cdot \nabla) \mathbf{V}_s]_r + \hat{r} \cdot \nabla \cdot \overleftrightarrow{\Pi}_s = n_s e_s (B_\phi V_{\theta s} - B_\theta V_{\phi s}) - p'_s, \quad (2)$$

where we note that the convective term has a contribution $-nmV_\theta^2/r$ which survives incompressibility and the flux-surface average. The RHS of the equation above is (minus the charge density times) what is called the observed radial electric field, while the LHS is usually neglected. Thus, when one measures a species' net toroidal and poloidal velocities (in addition to the usual density, temperature, and magnetic field measurements), one may determine that species' net radial viscous force.

III. DIRECT MEASUREMENT

The direct measurement of the radial electric field is accomplished by motional Stark effect diagnostics [4, 5] viewing the D_α line emission of injected neutrals excited by collisions with plasma particles. While it is claimed that the neutrals experience an electric field of $\mathbf{E}_{tot}^{neut} = \mathbf{V}^{neut} \times \mathbf{B} + \mathbf{E}_r$, one should note that the magnetic field in that equation should be evaluated in the frame moving with the neutrals [6] and that a radial electric field in the rest frame of the machine should also be modified by the neutrals' relative motion to appear as both electric and magnetic fields in its frame of reference. The measurements are biased towards observations along the midplane as a consequence of the neutral beam injection path and diagnostic port placement. Thus, the measurement of an electric field along the plasma midplane does not necessarily reveal the existence of a net radial electric field.

As Gauss' law is inviolate [1], any electric fields within a neutral, conducting medium necessarily are driven by changes in the magnetic flux density, giving us

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B}. \quad (3)$$

Examining the nonvanishing component of the curl of \mathbf{E} in (Z, R, ϕ) geometry,

$$[\nabla \times \mathbf{E}]_\phi = \frac{\partial}{\partial Z} E_R - \frac{\partial}{\partial R} E_Z = -\frac{\partial}{\partial t} B_\phi. \quad (4)$$

Thus, we observe that divergenceless electric fields in the (Z, R) plane may be generated by changes in the toroidal magnetic flux density. So, let's calculate the electric fields generated by a shifting of the toroidal magnetic field.

We suppose an initially uniform axial magnetic field and consider the effect of a rearrangement of flux as depicted in Figure 1, aligned to the vertical midplane, where the plasma is assumed to encompass totally the region under consideration and experiences no net change in magnetic flux. Noting the similarity between Faraday's law above and Ampere's law, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, let's us proceed with the calculation by treating each site experiencing a change in magnetic flux density as a line current generating a magnetic field, with an obvious difference in notation. The field generated at a perpendicular distance $\rho_p \equiv |\mathbf{r} - \mathbf{r}_p|$ from an infinite straight-line source p is simply $E_p = -\dot{B}_\phi^p / 2\pi\rho_p$, for $\dot{B} \equiv \partial B / \partial t$, thus the total field is

$$\mathbf{E}^0(\mathbf{r}) = \frac{1}{2\pi} \sum_p \frac{-\dot{B}_\phi^p}{\rho_p} \hat{\phi} \times \hat{\rho}_p. \quad (5)$$

We take the field so calculated over to toroidal geometry by applying the flux surface measure factor to the denominator, $\mathbf{E} = \mathbf{E}^0/(1 + \epsilon \cos \theta)$. For the flux displacement depicted, we get the electric field shown in Figure 2.

We verify our construction by evaluating the LHS of Equations (3), depicted in Figures 3 and 4. The spikes apparent in the divergence are not a worry, as the integral over each contribution vanishes to within machine precision, as does the net integral of the curl. The integral over each contribution to the curl returns the enclosed rate of flux shift, $-\dot{B}_\phi^p$, increased by a small factor over the straight-line source value as the source is no longer infinite—the electric field calculated is still valid, it just needs a little more source magnitude in toroidal geometry. Thus, we have determined the electric fields orthogonal to $\hat{\phi}$ generated by a vertical shift in the toroidal magnetic flux density. While simple, our model depicts the effect of relative motion between the flux surfaces for a magnetically confined plasma at semi-equilibrium, $\partial \mathbf{B}^{tot}/\partial t = 0$.

IV. CONCLUSIONS

As the net radial electrostatic field must vanish for a toroidally symmetric tokamak plasma at equilibrium, evaluation of a species' radial equation of motion most likely returns an estimate of that species' net radial viscous force. The requirement of no net electrostatic fields does not preclude the development of dynamic electric fields, as given by the construction above. Considering the relative motion of the flux surfaces at semi-equilibrium results in the generation of dynamic electric fields in the poloidal plane, which may account for the direct measurement thereof. Returning to our analogy with currents, two antiparallel line currents experience a mutually repulsive force, and so should our regions of flux shift—how that plays out in a magnetically confined plasma remains to be investigated.

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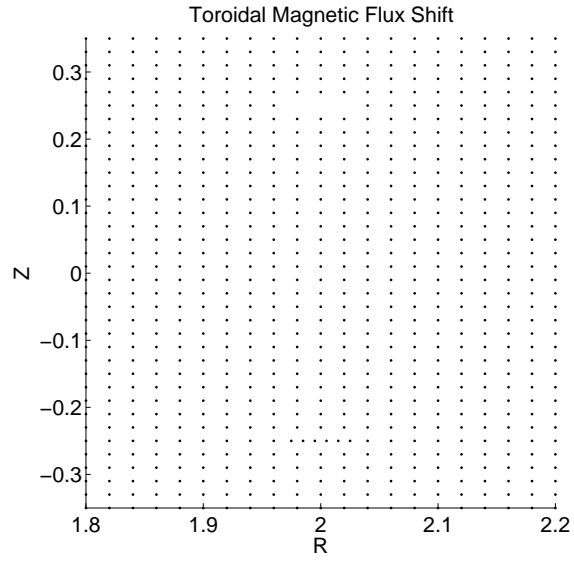


FIG. 1: Toroidal magnetic flux shift.

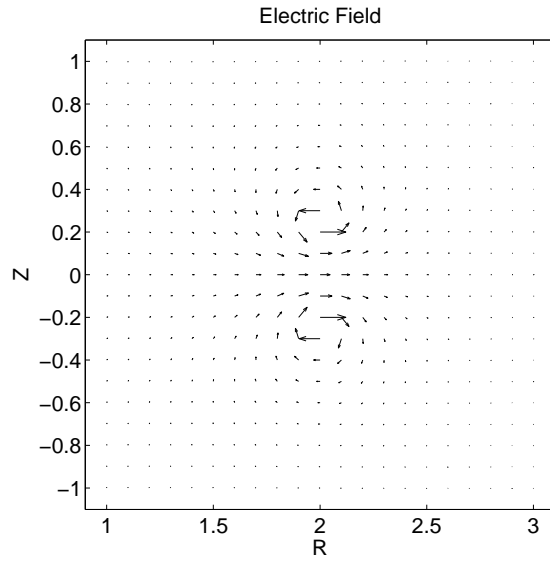


FIG. 2: Generated electric field.

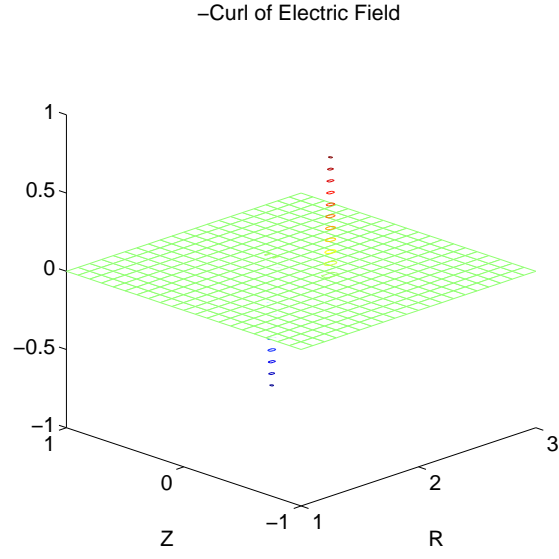


FIG. 3: Magnetic flux shift given by $-\nabla \times \mathbf{E}$.

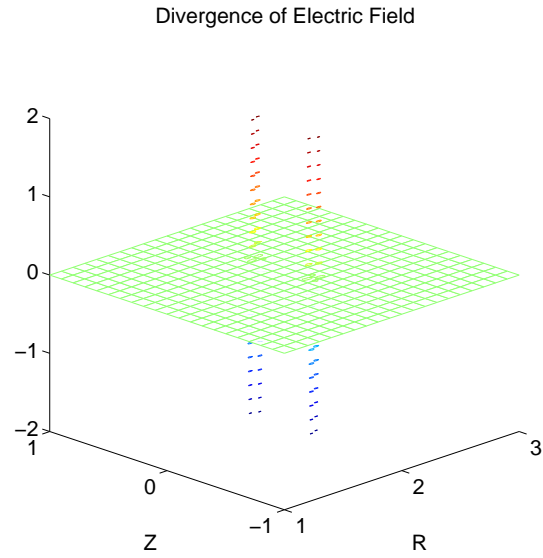


FIG. 4: Electric field divergence, $\int \nabla \cdot \mathbf{E} = 0$.